Parallels between Theory-Based Experiments in Political Science and Economics

Thomas Palfrey
Caltech

ESA
Tucson, AZ
11 11 11
Traditional View of Differences between Economics and Political Science

- Private vs. Public Decision Making
- Markets vs. Politics
- Money vs. Votes
- Transferable Utility vs. Non-transferable Utility
- Producers/Consumers vs. Candidates/Voters
- International Trade vs. International Relations
Parallel theoretical foundations

• The basic model of individual decision making is *homo economicus*
  – Bayes’ rule
• Selfish optimization

• The basic approach to predicting outcomes is based on equilibrium or stability.
  – The **core** as a predictive solution concept in competitive environments
  – Subgame perfect **Nash equilibrium** as a non-cooperative solution for games in extensive form.
  – Allows one to compare the effects of changing institutions (game forms)

• Can layer onto this basic approach nonstandard ‘behavioral assumptions’ for either political or economic models:
  – Disequilibrium approaches (e.g., level-k, adaptive learning)
  – Soft optimization (e.g., QRE)
  – Judgment fallacies (e.g., base-rate neglect, cursed equilibrium)
  – Social Preferences (e.g., altruism or inequality aversion, reciprocity)
Parallel Phenomena

• Auctions, markets, and voting institutions
  – Vote trading, logrolling, all pay auctions
• Competition
  – Candidates, Parties, Firms, Workers
• Monopoly and market power
  – Agenda control, dictatorship
• Information aggregation and rational expectations equilibrium
  – Polls, bandwagons
• Bargaining and compromise
  – Government formation, legislative bargaining, international conflict
• Public goods and free riding
  – Voter turnout, political participation, collective action
• Adverse selection in auctions and voting
  – Strategic voting and abstention, swing voter’s curse
• Mechanism Design
  – Comparative politics, voting rules, organizations

November 11, 2011
ESA Lecture. Tucson
Political Science Experiments5
Today’s focus: **Vote Trading**

Two papers:

“Competitive Equilibrium in Markets for Votes” with Alessandra Casella and Aniol Llorente-Saguer

“Vote Trading With and Without Party Leaders” with Alessandra Casella and Sébastien Turban.

The two papers demonstrate how auction and market models of economic exchange can be applied in a natural way to study political exchange.

Part of a larger research program on mechanism design in the context of political institutions.
Competitive Equilibrium in Markets for Votes

Alessandra Casella, Columbia University, NBER
Aniol Llorente-Saguer, Max Planck Institute, Bonn
Thomas R. Palfrey, Caltech

ESA Tucson Lecture
November 11, 2011
Majority Rule is widely used.
  ▶ Robustness to different informational settings.
  ▶ May’s Theorem (May, 1952)
  ▶ However… it **fails** to take into consideration **intensities**.
Majority Rule is widely used.
- Robustness to different informational settings.
- May’s Theorem (May, 1952)
- However... it fails to take into consideration intensities.

Example:
- Max, Major, and Eve need to decide on a binary issue.

<table>
<thead>
<tr>
<th></th>
<th>Alternative M</th>
<th>Alternative E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>
Majority Rule is widely used.

- Robustness to different informational settings.
- May’s Theorem (May, 1952)
- However... it fails to take into consideration intensities.

Example:

- Max, Major, and Eve need to decide on a binary issue.

<table>
<thead>
<tr>
<th></th>
<th>Alternative M</th>
<th>Alternative E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

- Majority rule outcome: M

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Majority Rule is widely used.

- Robustness to different informational settings.
- May’s Theorem (May, 1952)
- However... it **fails** to take into consideration **intensities**.

Example:

Max, Major, and Eve need to decide on a binary issue.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

- Majority rule outcome: M
- Efficient outcome: E
Possible solution... vote markets?

Distinction between:
- logrolling (pure exchange)
- **spot trading for money**.

- A **single** binary decision, made by majority rule.
- A **competitive market** for votes.
- Ignoring ethical concerns.
Conflicting claims:

1. A market for votes increases efficiency by expressing intensity.
2. Externalities might make a votes market inefficient.

But there is a prior issue one must address first:
What is the competitive equilibrium in a market for votes?

How does one apply general equilibrium theory to a model of political exchange?
A SIMPLE EXAMPLE
<table>
<thead>
<tr>
<th>Alternative M</th>
<th>Alternative E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
</tr>
<tr>
<td>Alternative M</td>
<td>Alternative E</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Max</td>
<td>10</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
</tr>
</tbody>
</table>

Existence Problem!

1. $p > 0$ and one voter has 3 votes. One vote is redundant. Excess supply.
2. $p > 0$ and one voter has 2 votes. One vote is worthless. Excess supply.
3. $p > 0$ and all have 1 vote. Either excess demand or supply.
4. $p = 0$. Excess demand.
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

▶ Votes are indivisible.
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings.
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings
- Public Goods

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings
- Public Goods
- No intrinsic value
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings
- Public Goods
- No intrinsic value
- Payoffs are discontinuous: Votes are worthless unless pivotal
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings
- Public Goods
- No intrinsic value
- Payoffs are discontinuous: Votes are worthless unless pivotal
- If one individual is pivotal then others are as well
There is no general model of decentralized vote markets for which a competitive equilibrium exists. These are pathological environments!

- Votes are indivisible.
- Externalities: Value of a vote depends on others’ holdings
- Public Goods
- No intrinsic value
- Payoffs are discontinuous: Votes are worthless unless pivotal
- If one individual is pivotal then others are as well
- Non-satiation is violated
In this paper, we:

- Propose the concept of Ex Ante Competitive Equilibrium.
- Apply it to a vote market.
- Prove existence.
- Characterize the equilibrium properties
- Test the equilibrium concept in a lab experiment.
The Environment

- A committee of \( N \) (odd) voters, deciding between two alternatives, \( A \) and \( B \).
The Environment

- A committee of \( N \) (odd) voters, deciding between two alternatives, \( A \) and \( B \).

- Each voter prefers \( A \) with probability \( \frac{1}{2} \) and is endowed with 1 indivisible vote.
The Environment

- A committee of $N$ (odd) voters, deciding between two alternatives, $A$ and $B$.
- Each voter prefers $A$ with probability $\frac{1}{2}$ and is endowed with 1 indivisible vote.
- Voter $i$ attributes value $v_i \in [0, 1]$ to his preferred alternative.
The Environment

- A committee of $N$ (odd) voters, deciding between two alternatives, $A$ and $B$.
- Each voter prefers $A$ with probability $\frac{1}{2}$ and is endowed with 1 indivisible vote.
- Voter $i$ attributes value $v_i \in [0, 1]$ to his preferred alternative.
- $v_i$ is privately known
The Decision Procedure

- Two stages:
  1. Voters can buy votes from each other using the numeraire
  2. Voters cast their votes, if any, for their preferred alternative
The Decision Procedure

- Two stages:
  1. Voters can buy votes from each other using the numeraire
  2. Voters cast their votes, if any, for their preferred alternative

- The decision is taken according to the majority of votes cast
The Decision Procedure

- Two stages:
  1. Voters can buy votes from each other using the numeraire
  2. Voters cast their votes, if any, for their preferred alternative

- The decision is taken according to the majority of votes cast
- Ties are resolved by a coin flip
Preferences and Demands

- Each voter’s net demand for votes belongs to the set $S_i = \{ s \in \mathbb{Z} | s \geq -1 \}$

- The set of feasible *mixed* demands of voter $i$ is the set of probability measures on $S_i$, denoted $\Sigma_i$
Preferences and Demands

- Each voter’s net demand for votes belongs to the set \( S_i = \{ s \in \mathbb{Z} | s \geq -1 \} \)

- The set of feasible *mixed* demands of voter \( i \) is the set of probability measures on \( S_i \), denoted \( \Sigma_i \)

- A rationing rule \( \mathcal{R} \) maps voters’ demands to a feasible allocation of votes \( \mathbf{X} = \{ \mathbf{x} \in \mathbb{Z}_+^n | \sum x_i = N, x_i \geq 0 \} \).
Preferences and Demands

- Each voter’s net demand for votes belongs to the set $S_i = \{ s \in \mathbb{Z} | s \geq -1 \}$

- The set of feasible mixed demands of voter $i$ is the set of probability measures on $S_i$, denoted $\Sigma_i$

- A rationing rule $\mathcal{R}$ maps voters’ demands to a feasible allocation of votes $X = \{ x \in \mathbb{Z}_+^n | \sum x_i = N, x_i \geq 0 \}$.

- Ex post utility equals: $u_i = v_i l_w - (x_i - 1) p$

- $U_i(\sigma, \mathcal{R}, p)$ is ex ante utility, given $p$, an action profile $\sigma$, and the rationing rule $\mathcal{R}$. 
The Equilibrium: Definition

- The set of demands $\sigma^*$ and price $p^*$ constitute an **Ex Ante Competitive Equilibrium** relative to rationing rule $\mathcal{R}$ if:

1. **Expected** utility maximization: For each agent $i$, $\sigma_i^*$ satisfies

   $$\sigma_i^* \in \arg \max_{\sigma_i \in \Sigma_i} U_i(\sigma_i, \sigma_{-i}^*, \mathcal{R}, p^*)$$
The Equilibrium: Definition

- The set of demands $\sigma^*$ and price $p^*$ constitute an **Ex Ante Competitive Equilibrium** relative to rationing rule $\mathcal{R}$ if:

1. **Expected** utility maximization: For each agent $i$, $\sigma_i^*$ satisfies

$$\sigma_i^* \in \arg \max_{\sigma_i \in \Sigma_i} U_i(\sigma_i, \sigma_{-i}^*, \mathcal{R}, p^*)$$

2. **Expected** market clearing: In expectation, the market clears:

$$\sum_{s \in S} q_{\sigma^*}(s) \sum_{i=1}^{n} s_i = 0$$
Proposition 1

- For all \( n \) and \( \{v_1, \ldots, v_n\} \) there exists an Ex Ante Vote-Trading Equilibrium with positive trade where
  - Voters 1 to \( n - 2 \) offer to sell their vote with probability 1;
  - Voters \( n - 1 \) and \( n \) demand \( \frac{n-1}{2} \) votes with probabilities \( \gamma_{n-1} \) and \( \gamma_n \) respectively, and offer their vote otherwise.

The paper considers one commonly-used rationing rule (all-or-nothing), but one can use other rules with similar results.
Equilibrium Properties

1. The only actions that in equilibrium are selected with positive probability are either $-1$ or $\frac{n-1}{2}$.

2. Any votes market outcome where trade occurs always results in dictatorship.

3. Rationing occurs with probability 1.
Return to simplified example, where sides are known. Similar equilibrium properties.

<table>
<thead>
<tr>
<th></th>
<th>Alternative M</th>
<th>Alternative E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

What is the ex ante equilibrium?
Return to simplified example, where sides are known. Similar equilibrium properties.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

What is the ex ante equilibrium?

1. \( p = v2/2 = 12/2 = 6 \)
Return to simplified example, where sides are known. Similar equilibrium properties.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

What is the ex ante equilibrium?

1. \( p = \nu 2/2 = 12/2 = 6 \)
2. Max’s demand: \( = -1 \) (supply one vote)
Return to simplified example, where sides are known. Similar equilibrium properties.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

What is the ex ante equilibrium?

1. \( p = \frac{v_2}{2} = \frac{12}{2} = 6 \)
2. Max’s demand: \( = -1 \) (supply one vote)
3. Eve’s demand: \( = 1 \) (demand one vote)
Return to simplified example, where sides are known. Similar equilibrium properties.

<table>
<thead>
<tr>
<th></th>
<th>Alternative M</th>
<th>Alternative E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Major</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Eve</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

What is the ex ante equilibrium?

1. \( p = v2/2 = 12/2 = 6 \)
2. Max’s demand: \( = -1 \) (supply one vote)
3. Eve’s demand: \( = 1 \) (demand one vote)
4. Major’s demand: mix equally between \(-1\) and \(1\)
For most parameters, only the second highest valuation voter mixes.
For most parameters, only the second highest valuation voter mixes.

For all $n$, there exists a finite threshold $\mu_n \geq 1$ such that if $v_n > \mu_n v_{n-1}$, then

- $\gamma_{n-1} = \frac{n-1}{n+1}$
- $\gamma_n = 1$
- $p = \frac{v_{n-1}}{n+1}$.

The parameters used in the experiment fulfill $v_n > \mu_n v_{n-1}$. 
How do vote markets compare to Majority Rule?
How do vote markets compare to Majority Rule?

Purely utilitarian welfare perspective

Assigning decision power to one voter with highest valuation can be welfare improving.
How do vote markets compare to Majority Rule?

Purely utilitarian welfare perspective

Assigning decision power to one voter with highest valuation can be welfare improving.
How do markets compare to Majority Rule?

From a purely utilitarian welfare perspective

- Assigning decision power to one voter with highest valuation can be welfare improving.
- Ignoring the will of all but highest valuation is unlikely to be desirable.
Proposition 2

For any sequence of well-behaved distributions $F_n$, there exists $\bar{n}$ such that if $n > \bar{n}$, $W_{MR} > W_{VM}$.
How can we test the theory?

In the field...

- We don’t know the preferences of voters/legislators
- Cannot directly control for other confounding factors such as budget constraints, influence of third parties, ...
- Illegal in many settings, so hard to find data
- One promising possibility: Voting shares of firms are bought and sold in financial markets.

In the lab instead...

- We control for preferences
- Avoid confounding factors
Experiments were conducted at Caltech (SSEL)

Interactions among subjects were computerized (Multistage)

”Naive subjects”. No subject participated in more than one session.
Experiments were conducted at Caltech (SSEL)

Interactions among subjects were computerized (Multistage)

”Naive subjects”. No subject participated in more than one session.

8 sessions were run in total, 4 with $N = 5$ and 4 with $N = 9$. 
Subjects were randomly assigned to be in favor of X or Y.
They were assigned privately known valuations $v_i \in (0, 1000]$
All subjects were endowed with one vote.
Continuous multiunit open book market.
**Standard Trading Mechanism:** Multiple-Unit Open-Book Computerized Double Auction

- Orders (bids or offers) allowed at any time, for one or multiple votes.
- New orders did not cancel outstanding orders. But orders could be canceled at any time.
- Computer screens showed and updated (i) outstanding orders; (ii) the number of votes held by each subject.
- Initial loan of 10,000 points. Cash holdings were updated and no purchase with negative balance was allowed.
- Subjects could not short-sell votes.
Standard Trading Mechanism: Multiple-Unit Open-Book Computerized Double Auction

- Orders (bids or offers) allowed at any time, for one or multiple votes.
- New orders did not cancel outstanding orders. But orders could be canceled at any time.
- Computer screens showed and updated (i) outstanding orders; (ii) the number of votes held by each subject.
- Initial loan of 10,000 points. Cash holdings were updated and no purchase with negative balance was allowed.
- Subjects could not short-sell votes.
- 2 minute trading stage (continuous double auction)
- After the trade stage, committee decision by majority rule.
Parameters and design:

- Each committee traded in four different markets in different sequences.
- Each market was repeated five times.
- A 2 x 2 set of valuations: \( \{ \text{High, Low} \} \times \{ \text{Top, Bottom} \} \).
- \( \{ \text{High, Low} \} \) refers to the value of \( v_{n-1} \).
- \( \{ \text{Top, Bottom} \} \) refers to skewed up or down (same \( v_{n-1} \)).
Design of sessions

<table>
<thead>
<tr>
<th>Session</th>
<th>Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HT</td>
</tr>
<tr>
<td>2</td>
<td>LT</td>
</tr>
<tr>
<td>3</td>
<td>HB</td>
</tr>
<tr>
<td>4</td>
<td>LB</td>
</tr>
<tr>
<td>1</td>
<td>LB</td>
</tr>
<tr>
<td>2</td>
<td>HB</td>
</tr>
<tr>
<td>3</td>
<td>LT</td>
</tr>
<tr>
<td>4</td>
<td>HT</td>
</tr>
<tr>
<td>1</td>
<td>HB</td>
</tr>
<tr>
<td>2</td>
<td>LT</td>
</tr>
<tr>
<td>3</td>
<td>HT</td>
</tr>
<tr>
<td>4</td>
<td>LB</td>
</tr>
<tr>
<td>1</td>
<td>LB</td>
</tr>
<tr>
<td>2</td>
<td>HT</td>
</tr>
<tr>
<td>3</td>
<td>LT</td>
</tr>
<tr>
<td>4</td>
<td>HB</td>
</tr>
</tbody>
</table>

- Four sessions with $n = 5$, four sessions with $n = 9 \Rightarrow 56$ subjects.
- Caltech undergraduates; June 2009.
- Average length: 75 minutes, average payoff: $29.$
Design

- Transaction Prices
- Vote allocations
- Welfare

Experiments

- Committees traded during four different matches
- Each match had different market parameters
- Five replays of each match for convergence
- Values fixed across replays; sides reshuffled

Introduction

The Model

Theoretical Results

Casella, Llorente-Saguer & Palfrey

Competitive Equilibrium in Markets for Votes
Design
Transaction Prices
Vote allocations
Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Introduction
The Model
Theoretical Results
Experiments
Conclusions
Decentralized trading vs. Party leadership
Conclusions

Design
Transaction Prices
Vote allocations
Welfare

Committees traded during four different matches
Each match had different market parameters
Five replays of each match for convergence
Values fixed across replays; sides reshuffled

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Decentralized trading vs. Party leadership

Experiments
- Committees traded during four different matches
- Each match had different market parameters
- Five replays of each match for convergence
- Values fixed across replays; sides reshuffled
Decentralized trading vs. Party leadership

Experiments

Design
Transaction Prices
Vote allocations
Welfare

Competitive Equilibrium in Markets for Votes
Committees traded during four different matches
Each match had different market parameters
Five replays of each match for convergence
Values fixed across replays; sides reshuffled
Committees traded during four different matches.
Each match had different market parameters.
Five replays of each match for convergence.
Values fixed across replays; sides reshuffled.

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Decentralized trading vs. Party leadership

Conclusions

Design
Transaction Prices
Vote allocations
Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Decentralized trading vs. Party leadership

Design
- Transaction Prices
- Vote allocations
- Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Introduction
The Model
Theoretical Results
Experiments
Conclusions
Decentralized trading vs. Party leadership
Conclusions

Design
Transaction Prices
Vote allocations
Welfare

Committees traded during four different matches
Each match had different market parameters
Values fixed across replays; sides reshuffled

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Design
Transaction Prices
Vote allocations
Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Design
Transaction Prices
Vote allocations
Welfare

Introduction
The Model
Theoretical Results
Experiments
Conclusions
Decentralized trading vs. Party leadership
Conclusions

Design
Transaction Prices
Vote allocations
Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
 Committees traded during four different matches
Committees traded during four different matches
Each match had different market parameters
Committees traded during four different matches
Each match had different market parameters
Five replays of each match for convergence
▶ Committees traded during four different matches
▶ Each match had different market parameters
▶ Five replays of each match for convergence
▶ Values fixed across replays; sides reshuffled
9LT Market. Valuations range from 105 to 753; the equilibrium price is 50.
Introduction
The Model
Theoretical Results
Experiments
Conclusions

Decentralized trading vs. Party leadership

Conclusions

Design
Transaction Prices
Vote allocations
Welfare

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Key findings about pricing:

▶ Prices generally start above the ex ante equilibrium price.
▶ Prices decline over time within a match.
▶ Overpricing declines across matches.
▶ Convergence toward equilibrium prices from above.
How to explain high prices?
Many possibilities.
We explore one:

Risk Aversion

Proposition

Suppose $u(\cdot) = -e^{-\rho(\cdot)}$ with $\rho > 0$, and $R_1$ is the rationing rule. Then for all our experimental treatments the set of strategies in Theorem 1 together with the price $p = \frac{2}{\rho}(n+1)\ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho}v_n - \frac{1}{2}\right)$ constitute an Ex Ante Vote-Trading Equilibrium.
How to explain high prices?

Many possibilities.

We explore one:

Risk Aversion.
How to explain high prices?
Many possibilities.
We explore one:
Risk Aversion.

Risk aversion

**Proposition**

Suppose \( u(\cdot) = -e^{-\rho(\cdot)} \) with \( \rho > 0 \), and R1 is the rationing rule. Then for all our experimental treatments the set of strategies in Theorem 1 together with the price \( p = \frac{2}{\rho(n+1)} \ln \left( \frac{1}{2} + \frac{1}{2} e^{\rho v n^{-1}} \right) \) constitute an Ex Ante Vote-Trading Equilibrium.
### Transaction prices from last match and round

<table>
<thead>
<tr>
<th>N</th>
<th>Last Round Average</th>
<th>HB</th>
<th>HT</th>
<th>LB</th>
<th>LT</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Last Round Average</td>
<td>240</td>
<td>258</td>
<td>150</td>
<td>81.5</td>
</tr>
<tr>
<td></td>
<td>RN Equilibrium</td>
<td>150.5</td>
<td>150.5</td>
<td>83.5</td>
<td>83.5</td>
</tr>
<tr>
<td></td>
<td>RA Equilibrium</td>
<td>301</td>
<td>301</td>
<td>167</td>
<td>167</td>
</tr>
<tr>
<td>9</td>
<td>Last Round Average</td>
<td>176.4</td>
<td>163</td>
<td>102</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>RN Equilibrium</td>
<td>90.3</td>
<td>90.3</td>
<td>50.1</td>
<td>50.1</td>
</tr>
<tr>
<td></td>
<td>RA Equilibrium</td>
<td>180.6</td>
<td>180.6</td>
<td>100.2</td>
<td>100.2</td>
</tr>
</tbody>
</table>
Testing convergence to equilibrium:

\[
\text{Asymptotic Regression (a la Noussair-Plott-Riezman)}
\]

\[
\dot{p}_M = \alpha M + \beta M_m (1/t) + \epsilon_{Mm}
\]

\(t\) is the unit of time in the experiment (seconds)

\(M\) is the index for Market

\(m\) is the index for match.

\(\alpha\) constrains each match in a market (each color in the plots) to converge to the same price

\(\beta\) allows convergence path to differ across matches.
Testing convergence to equilibrium: Asymptotic Regression (a la Noussair-Plott-Riezman)

\[ p_{Mt} = \alpha_M + \beta_{Mm}(1/t) + \varepsilon_{Mmt} \]

- \( t \) is the unit of time in the experiment (seconds)
- \( M \) is the index for Market
- \( m \) is the index for match.
- \( \alpha_M \) is the asymptote of price \( p_M \)
- constrains each match in a market (each color in the plots) to converge to the same price
- allows convergence path to differ across matches.
### Table: Linear regression of the log of the realized price

<table>
<thead>
<tr>
<th></th>
<th>$n = 5$</th>
<th>$n = 9$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\alpha}_M$</td>
<td>s.e</td>
<td>95% Conf. Interval</td>
<td>$\log(\hat{p}^m)$</td>
</tr>
<tr>
<td>HT</td>
<td>5.57</td>
<td>0.20</td>
<td>[4.94, 6.20]</td>
<td>[5.01, 5.71]</td>
</tr>
<tr>
<td>HB</td>
<td>5.46</td>
<td>0.16</td>
<td>[4.94, 5.97]</td>
<td>[5.01, 5.71]</td>
</tr>
<tr>
<td>LT</td>
<td>5.18</td>
<td>0.20</td>
<td>[4.54, 5.82]</td>
<td>[4.42, 5.12]</td>
</tr>
<tr>
<td>LB</td>
<td>4.89</td>
<td>0.29</td>
<td>[3.97, 5.81]</td>
<td>[4.42, 5.12]</td>
</tr>
<tr>
<td></td>
<td>5.35</td>
<td>0.11</td>
<td>[5.00, 5.69]</td>
<td>[4.50, 5.20]</td>
</tr>
<tr>
<td>HB</td>
<td>5.35</td>
<td>0.095</td>
<td>[5.05, 5.66]</td>
<td>[4.50, 5.20]</td>
</tr>
<tr>
<td>LT</td>
<td>5.09</td>
<td>0.15</td>
<td>[4.62, 5.57]</td>
<td>[3.91, 4.61]</td>
</tr>
<tr>
<td>LB</td>
<td>4.94</td>
<td>0.04</td>
<td>[4.82, 5.06]</td>
<td>[3.91, 4.61]</td>
</tr>
</tbody>
</table>

**Casella, Llorente-Saguer & Palfrey**

*Competitive Equilibrium in Markets for Votes*
Average final vote allocations. Theory vs. Data?
Casella, Llorente-Saguer & Palfrey

Competitive Equilibrium in Markets for Votes
How frequent is **dictatorship**?

<table>
<thead>
<tr>
<th>Market</th>
<th>n</th>
<th>HB</th>
<th>HT</th>
<th>LB</th>
<th>LT</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>55</td>
<td>45</td>
<td>100</td>
<td>65</td>
<td>66.25</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>25</td>
<td>5</td>
<td>20</td>
<td>0</td>
<td>12.50</td>
</tr>
</tbody>
</table>

- More frequent in $B$ treatments.
- More frequent in with $n = 5$.
- More frequent with experience.
- Note: dictatorship with $n = 9$ means buying 4 votes.
Ex ante efficiency rates:

<table>
<thead>
<tr>
<th>Market</th>
<th>LB</th>
<th>LT</th>
<th>HB</th>
<th>HT</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=5 Realized</td>
<td>0.85</td>
<td>0.69</td>
<td>0.85</td>
<td>0.73</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.84</td>
<td>0.71</td>
<td>0.78</td>
<td>0.62</td>
</tr>
<tr>
<td>Majority Rule</td>
<td>0.79</td>
<td>0.90</td>
<td>0.83</td>
<td>0.97</td>
</tr>
<tr>
<td>N=9 Realized</td>
<td>0.83</td>
<td>0.64</td>
<td>0.80</td>
<td>0.72</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.75</td>
<td>0.62</td>
<td>0.70</td>
<td>0.53</td>
</tr>
<tr>
<td>Majority Rule</td>
<td>0.74</td>
<td>0.89</td>
<td>0.79</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Realized Efficiency > Equilibrium Efficiency.
Ex ante efficiency rates:

<table>
<thead>
<tr>
<th>Market</th>
<th>LB</th>
<th>LT</th>
<th>HB</th>
<th>HT</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=5 Realized</td>
<td>0.85</td>
<td>0.69</td>
<td>0.85</td>
<td>0.73</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.84</td>
<td>0.71</td>
<td>0.78</td>
<td>0.62</td>
</tr>
<tr>
<td>Majority Rule</td>
<td>0.79</td>
<td>0.90</td>
<td>0.83</td>
<td>0.97</td>
</tr>
<tr>
<td>N=9 Realized</td>
<td>0.83</td>
<td>0.64</td>
<td>0.80</td>
<td>0.72</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>0.75</td>
<td>0.62</td>
<td>0.70</td>
<td>0.53</td>
</tr>
<tr>
<td>Majority Rule</td>
<td>0.74</td>
<td>0.89</td>
<td>0.79</td>
<td>0.93</td>
</tr>
</tbody>
</table>

- Realized Efficiency > Equilibrium Efficiency.
- B (T): Realized efficiency > (<) Majority Rule efficiency.
Conclusions: Theory

- General Equilibrium Analysis can be applied to vote trading
  - Equilibrium Existence Theorem
  - Dictator
  - Inefficiency
  - Comparative statics
Conclusions: Experiments

- Decentralized vote trading
  - Prices converge from above to the competitive equilibrium price range
  - Average vote allocations close to equilibrium
  - Frequent dictators, especially in small committees
  - Inefficiencies can result from too much trading. Externalities
  - Efficiency and comparative statics consistent with general equilibrium theory
Second Study: Partisanship public information. Decentralized trading vs. Party leadership (with Casella and Turban)

- Existence of ex ante competitive equilibrium with decentralized trading with public partisanship
- Need a different model for centralized trading: Bilateral exchange with private values. Bayesian game.
The Environment

- A committee of size $n$, each individual has one vote.
- Two alternatives, $X$ and $Y$.
- It is publicly known that $M$ individuals prefer $X$, $m \leq M$ prefer $Y$.
- Values $v_i \in [0, 1]$, privately known.
- It is known that $v_i$ is iid $\sim F(v)$ atomless and will full support $[0, 1]$. 
The Decision Procedure

- Two stages:
  1. Leaders (C) or individual voters (U) can buy votes from each other using the numeraire;
  2. Leaders (C) or individual voters (U) cast their votes, if any, for their preferred alternative.

- The decision is taken according to the majority of votes cast.
- Ties are resolved by a coin flip.
Coordinated trade:

- The leader internalizes the total value of the group and enforces transfers, if necessary.
- Ex-post utility is given by:

\[ u_i^C = \sum_{j \in G_i} v_j l_w - x_i^C p \quad \text{where} \quad x_i^C \in \{-G_i, ..., G_{-i}\}. \]

Uncoordinated trade:

- Each individual trades independently.
- Ex-post utility is given by:

\[ u_i^U = v_i l_w - x_i^U p \quad \text{where} \quad x_i^U \in \{-1, ..., n-1\}. \]
Mechanism Design Theory: (Myerson and Satterthwaite; Cramton, Gibbons and Klemperer). Suppose vote trading is coordinated by party leaders. Then an efficient, incentive compatible, and interim individually rational trading mechanism exists if and only if \( M = m \). If \( M \neq m \), the optimal mechanism has too little trade: the majority wins too often.
Coordinated trading

1. Two equal-sized groups.

- Each ”owns half the decision”.
- In the lab. Bid-only continuous open book auction.
- Equivalent to a second-price auction: the winner pays loser the loser’s bid.

\[ u_i = v_i - b_j \text{ if } b_i > b_j \]
\[ u_i = b_i \text{ if } b_j > b_i \]

Equilibrium bid function: \( b_i = \frac{1+2v_i}{6} \)

- Efficient, incentive compatible, and interim individually rational.
- Identical to a CGK efficient mechanisms
2. **Two unequal-sized groups.**

- The majority "owns" the decision (seller). Minority is the buyer.
- In the lab: trade only if $b_m > v_M$.
- Theory: A sealed buyer's bid double auction (Satterthwaite and Williams, 1989).
  - $u_s = v_s$ if $b_s > b_b$
  - $u_s = b_b$ if $b_s < b_b$
  - $u_b = 0$ if $b_s > b_b$
  - $u_s = v_b - b_b$ if $b_s < b_b$
  - Equilibrium: $b_s = v_s$. $b_b = \frac{v_b}{2}$
  - Incentive compatible and interim individually rational.
  - Inefficient: trade only if $v_b > 2v_s$. 

---

Casella, Llorente-Saguer & Palfrey  Competitive Equilibrium in Markets for Votes
Decentralized trading vs. Party leadership

Casella, Llorente-Saguer & Palfrey

Competitive Equilibrium in Markets for Votes
Decentralized Trade

- Use general equilibrium market model: Ex ante Competitive Equilibrium
- Equilibrium with positive price and positive trade exists under weak assumptions, even when partisanship is known
- Generally results in minority winning too often
Summary

- Coordinated trade with equal groups: fully efficient
- Coordinated trade with unequal groups: the majority wins too often
- ...but superior to voting without vote trading
- Uncoordinated market trade with unequal groups: the minority wins too often
- ...and inferior to voting without vote trading
Experimental Design

- One treatment per session; 25 rounds plus 1 practice round.
- Subjects matched randomly in committees, each choosing between $X$ and $Y$.
- It was common knowledge that $m$ subjects favored $Y$ and $M$ favored $X$.
- Values were drawn randomly by the computer from a Uniform on $[1, 100]$.
- Values were private information.
- All subjects had one initial vote and 200 points (to be paid back).
- Any subject could post a bid for a vote.
- The bid appeared on all monitors, together with the bidder’s group and a tally of each group’s total votes.
- The market was open for 3 minutes.
Treatments:

1, 1 (party leaders)  3, 2C (party leaders)  3, 2 (decentralized)

- In 1,1 and 3,2C: 2 subjects with opposite preferences.
- In 1,1: one vote each.
- In 3,2C: one subject had 3 votes and one had 2.
- In both cases: each subject value was a random draw from a Uniform over [1, 100].
Main Findings:

- Minority wins too much with decentralized trade (3,2); too little with party leaders (3,2C)
- Prices generally above the equilibrium.
- Convergence downward to equilibrium prices except for 3,2C.
- Centralized vote trading leads to efficiency gains.
- Decentralized vote trading leads to efficiency losses.
- More trade in 1,1 (66%) than 3,2C (59%). But less than 100% efficiency.
Convergence downward to equilibrium prices except for 3,2C.
Convergence downward to equilibrium prices except for 3,2C
Realized Gains from trade in 3,2C
Realized Gains from trade in 1,1
Surplus: Majority Rule vs. Vote Trading

Realized v. Majority-rule Payoffs

Rounds 11-25

Casella, Llorente-Saguer & Palfrey
Competitive Equilibrium in Markets for Votes
Minority Victories: Decentralized vs. Centralized

Realized v Predicted Minority Victories
Rounds 11-25

Realized v Efficient Minority Victories
Rounds 11-25

Competitive Equilibrium in Markets for Votes
Conclusions: Theory

- General Equilibrium Analysis can be applied to vote trading
  - Equilibrium Existence Theorem
  - Dictator
  - Inefficiency
  - Comparative statics
Conclusions: Theory

- General Equilibrium Analysis can be applied to vote trading
  - Equilibrium Existence Theorem
  - Dictator
  - Inefficiency
  - Comparative statics

- Mechanism design theory can be applied to centralized vote trading
  - Results depend on relative size of group
  - Equal size groups: CGK applies: first best possible
  - Unequal size parties: M-S applies: inefficiency unavoidable. Majority wins too often.
Conclusions: Experiments

- Decentralized vote trading
  - Prices converge from above to the competitive equilibrium price range
  - Average vote allocations close to equilibrium
  - Frequent dictators, especially in small committees
  - Inefficiencies can result from too much trading. Externalities
  - Efficiency and comparative statics consistent with general equilibrium theory
Conclusions: Experiments

- Decentralized vote trading
  - Prices converge from above to the competitive equilibrium price range
  - Average vote allocations close to equilibrium
  - Frequent dictators, especially in small committees
  - Inefficiencies can result from too much trading. Externalities
  - Efficiency and comparative statics consistent with general equilibrium theory

- Centralized vote trading between party leaders
  - Significant efficiency gains when parties are equal size
  - Smaller, insignificant efficiency gains when parties are unequal in size
  - Inefficiencies result from too little vote trading.
Parallel Phenomena

- Auctions, markets, and voting institutions
  - Vote trading, logrolling, all pay auctions
- Competition
  - Candidates, Parties, Firms, Workers
- Monopoly and market power
  - Agenda control, dictatorship
- Information aggregation and rational expectations equilibrium
  - Polls, bandwagons
- Bargaining and compromise
  - Government formation, legislative bargaining, international conflict
- Public goods and free riding
  - Voter turnout, political participation, collective action
- Adverse selection in auctions and voting
  - Strategic voting and abstention, swing voter’s curse
- Mechanism Design
  - Comparative politics, voting rules, organizations